

# A Hagedorn temperature in post-inflationary dynamics?

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We consider the possibility of regarding the maximum temperature  $T_{\max}$  of a plasma of inflaton decay products as a Hagedorn temperature connected with the transition from an inflaton-dominated epoch to a radiation-dominated universe. Employing a “bottom-up” approach to inflaton decay, we show that the limiting temperature  $T_H \sim m_\phi$ , as obtained from a numerical analysis of the evolution equations, can also be understood theoretically in a Hagedorn picture of the phase transition.

The concept of Hagedorn temperature [1] in statistical systems is commonly invoked as a scale that signals the onset of a phase transition [2]. While it does not directly relate to the usual thermodynamic definition of temperature, it can be regarded as a limiting temperature where the population of energy states begins to exceed the Boltzmann weight, implying that the original degrees of freedom strongly overlap in momentum space and lose their identity. One example of this is the melting of hadronic and glueball states in relativistic heavy-ion collisions, where the Hagedorn temperature  $T_H$  can be derived from the Bag model [3] or a stringy model [4] of hadrons, and can be used as an estimate of the deconfinement temperature, though the exact equivalence of the two is hard to establish, at least in four-dimensional gauge theories [5]. However, the idea of a number density of states that grows exponentially with temperature is quite natural coming from a theory with confining forces (like Regge theory), and may be useful when the exact nature of these microscopic forces is far from clear. A closely related example is Hagedorn-type behavior in some string theories at weak coupling [2]. In the context of early string dominated cosmology, a Hagedorn-type phase could be associated with open strings attached to D-branes which has recently been shown to provide a negative pressure necessary to drive inflation at high temperatures near the string scale [6]. This is accompanied by an exponential growth in the number of string modes with energy.

In this letter, we explore the possibility to have a Hagedorn limit of a quite different nature in cosmology, namely, in post-inflationary dynamics that culminate in reheating of the universe. Inflation is a well known contender for making the Universe flat, besides generating adiabatic density perturbations with an almost flat spectrum [7]. However, inflation must end in order to provide a thermal bath of radiation in order to facilitate synthesis of light nuclei. The standard paradigm is that the inflaton field,  $\phi$ , enters the oscillatory phase and decays perturbatively into radiation quanta, which subsequently thermalize completely through interactions, yielding a reheat temperature  $T_{\text{reh}}$ , when the universe is completely radiation dominated [8]. It has also been proposed that

an initial non-perturbative decay of the inflaton, dubbed as preheating, is a plausible scenario [10]. Nevertheless there has to be a subsequent stage of thermalization after preheating quite similar to perturbative reheating [9]. Yet another interesting possibility is to have a non-topological defect formation from the fragmentation of the inflaton condensate, such as  $Q$ -balls [11], where reheating occurs via surface evaporation of  $Q$ -balls.

Irrespective of a particular scenario the final reheating temperature plays a very crucial role in the early universe, for e.g. see [12]. The reheat temperature can be constrained by bounds coming from thermal and non-thermal gravitino production during reheating/preheating in supersymmetric theories [13]. Typically,  $T_{\text{reh}} < 10^9$  GeV is required, for gravitino mass  $m_{3/2} \sim 100$  GeV, in order to avoid constraints coming from successful predictions of Big Bang Nucleosynthesis (BBN), for e.g. see [14].

The reheat temperature is not, however, the maximum temperature of a thermal plasma, which may be two orders of magnitude higher (but less than  $m_\phi$ ) [8, 15]. Physically, this maximum temperature, denoted by  $T_{\max}$ , arises due to the competition between early entropy production within light relativistic degrees of freedom at the earliest stages of inflaton decay and continued expansion of the universe which dilutes the energy density. The main objective of this work is to show that  $T_{\max}$  may be regarded as a limiting temperature, if we trace the inflaton decay process backwards in time, up to a point when coherent inflaton production from the radiation fields begins. We take into account thermal factors that capture the essential feature of Bose enhancement when the inflaton population builds up. We support our theoretical conjecture about this limiting temperature as a Hagedorn temperature with a numerical analysis of the evolution equations for matter (inflaton) and radiation energy densities run backwards in time.

After inflation ends, the energy density stored in coherent oscillations of the inflaton field is transferred to radiation quanta over a period of time. Several features of this transition, such as thermalization [9], associated heavy-particle production [15], and non-perturbative processes (preheating) have been widely investigated [10, 11].

However, little is known about the dynamical process that results in inflaton decay. We may invert this problem, and ask the question as to how the inflaton field may be generated coherently from the radiation fields? Although the coupling between the two is small, our view is that Bose enhancement of the inflaton in the final state can lead to coherence, similar to a condensate effect. Incorporating this feature in the Boltzmann equations that describe the perturbative decay of the inflaton is not hard. We may run the system of equations (see below) backwards in time (decreasing scale factor) to study how the energy densities evolve. We show that an exponential enhancement of the inflaton density can overcome the Boltzmann weight, very similar to the Hagedorn picture. The temperature at this instant is the limiting temperature, beyond which the radiation energy density drops rapidly to zero. This may be interpreted as  $T_{\text{max}}$ , the maximum or limiting temperature. Thus, we have an alternate physical description of the maximum temperature, and we will quantify it both theoretically and numerically in following.

We consider the interaction  $L_{int} = g\phi\bar{f}f$  where  $f$  denotes a fermion<sup>1</sup>. The evolution equation for the inflaton energy density  $\rho_\phi$  is given by

$$\dot{\rho}_\phi + 3H\rho_\phi = \Gamma_\phi\rho_\phi(1 + n_B(m_\phi)), \quad (1)$$

$$\Gamma_\phi = \frac{g^2 m_\phi}{32\pi} = \alpha_\phi m_\phi, \quad (2)$$

where  $H$  is the Hubble expansion rate and  $\Gamma_\phi$  is the inflaton decay rate. Note that the source term for  $\rho_\phi$  is enhanced by the Bose factor  $(1 + n_B(m_\phi)) = 1/(1 - e^{-m_\phi/T})$ , since the inflaton is a scalar field. Since the expression for  $\Gamma_\phi$  is derived for zero total momentum (center-of-mass frame), the  $p = 0$  mode of the inflaton is being populated.  $\rho_\phi$  as defined here is the *thermal* energy density

$$\rho_\phi = \frac{m_\phi^4}{(e^{m_\phi/T} - 1)}. \quad (3)$$

The thermal factor comes out naturally when one considers fermions described by a thermal distribution fusing to form the inflaton. The analogous equation for  $\rho_R$ , the energy density of radiation is

$$\dot{\rho}_R + 4H\rho_R = -\Gamma_\phi\rho_\phi(1 + n_B(m_\phi)). \quad (4)$$

We evolve eqns. (1) and (4) numerically, for which purpose it is convenient to define rescaled quantities as

$$x = a/m_\phi, \quad \Phi = \rho_\phi a^3, \quad R = \rho_R a^4/m_\phi. \quad (5)$$

where  $a$  is the scale factor of the Friedman-Robertson-Walker metric. In terms of rescaled quantities, the evolution equations take the simple form

$$\left(\frac{d\Phi}{dx}\right) = c \frac{x\Phi}{\sqrt{x\Phi + R}}(1 + n_B(x, R)), \quad (6)$$

$$\left(\frac{dR}{dx}\right) = -c \frac{x^2\Phi}{\sqrt{x\Phi + R}}(1 + n_B(x, R)). \quad (7)$$

where  $c = \sqrt{\frac{3}{8\pi}}\alpha_\phi M_{Pl} m_\phi^{5/2}$ . These equations are similar in form to those in [15], but the rescaling is different since we wish to study the build-up rather than the decay of the inflaton, for which time runs backwards. Furthermore, we include the Bose enhancement factor that was not required in [15]. For the numerical analysis, we choose  $m_\phi = 10^{13}$  GeV,  $T_{\text{reh}} \sim 10^9$  GeV. The traditional expression for the reheat temperature corresponding to a thermal bath dominated solely by relativistic particles is given by [8]

$$T_{\text{reh}} = 0.2 \left(\frac{200}{g_*}\right)^{1/4} \sqrt{\alpha_\phi m_\phi M_{Pl}}, \quad (8)$$

where  $g_*$  accounts for the relativistic degrees of freedom in the plasma. Using this relation, and with  $g_* = 200$ , we obtain  $\alpha_\phi = 2.3 \times 10^{-13}$  and  $c = 8.7 \times 10^{37.5}$ . We begin the evolution from  $T_{\text{reh}} \sim 10^9$  GeV, which fixes the radiation energy density through the Stefan-Boltzmann law

$$\rho_R = \left(\frac{g_*\pi^2}{30}\right)^{1/4} T^4 \approx 2.85 \times 10^{36} \text{ GeV}^4 \quad (9)$$

However note that in our case there is no inflaton oscillation dominated phase and radiation is the only component we start with. In this respect the traditional notion of reheat temperature, which applies to a largest temperature of a thermal bath only dominated by relativistic particles, does not hold in this scenario. Nevertheless the numerical significance of eqn. (8) continues to be important as an estimation of a temperature of a relativistic bath when the entire inflaton quanta has been converted into radiation.

The qualitative features of the result of the numerical analysis are independent of the choice of initial value of scale factor  $a$ , although  $x, \Phi, R$  are quantitatively dependent on it. We choose  $a = 10^8$  ( $x = 10^{-5}$ ) at the beginning, and the results of the evolution across nearly 4 e-foldings are displayed in Fig. 1.

While the radiation curve  $R(x)$  (long dashed line) is remaining constant, the temperature  $T \propto a^{-1}$ , as expected for the radiation-dominated epoch. This is because both  $\rho_R$  and  $a$  are changing. As the inflaton energy density (solid line) builds up, the radiation energy density drops rapidly to zero. At this point, it is easy to see from eqn.(6) that  $\Phi(x) \propto x^3$ , which is the usual initial condition obtained in standard evolution of the inflaton decay

<sup>1</sup> For Standard model quarks or leptons, such an interaction cannot be written down since it would not be a  $SU(2)$  gauge invariant monomial. For the sake of exemplification, we assume that these fermions are pure singlet.

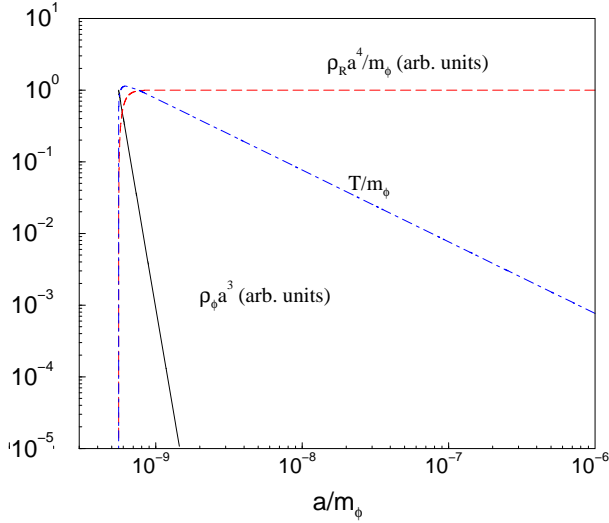


FIG. 1: Evolution (backwards in time or with decreasing scale factor) of the inflaton and radiation energy densities. The value of the limiting temperature is at the peak  $T_{\max} \approx m_\phi$ .

equation. Physically, our picture is quite different from that in [15], in that we can interpret  $T_{\max}$  as a limiting temperature, where the inflaton fields acquires a coherent nature, and the phase transition to an inflaton-dominated universe is underway. This interpretation, though novel, can be supported by a simple analytical estimate of  $T_{\max}$  which we presently derive.

It is also interesting to observe that in our case the process of thermalization is actually occurring very close to  $T_{\max}$  in such a way that entropy density of a radiation dominated bath follows an adiabatic expansion law  $sa^3 = \text{constant}$ , where  $s$  is the entropy density. In this respect we are tempted to associate  $T_{\max}$  as an actual reheat temperature of the universe.

At the limiting temperature, when the inflaton energy density is dominant,  $\Gamma_\phi \gg H$  holds, and we may neglect the  $3H\dot{\rho}_\phi$  term in eqn.(1). Assuming the temperature to be approximately constant over a small range of time, we may integrate the equation from time  $t_0$  to  $t$  ( $t_0 < t$  in the absolute sense) to obtain

$$\ln\left(\frac{n_\phi(t)}{n_\phi(t_0)}\right) = \frac{\alpha_\phi m_\phi (t - t_0)}{1 - e^{-m_\phi/T}}, \quad (10)$$

where  $n_\phi = \rho_\phi/m_\phi$ . This equation governs the rate of increase in the inflaton number density in a small interval  $t - t_0$  near the transition. This time interval must be of the order of (or slightly less than)  $1/\Gamma_\phi \lesssim 1/(\alpha_\phi m_\phi)$ , because subsequently, the inflaton density drops off much more rapidly than the temperature. This means we may approximate as

$$n_\phi(t) \approx n_\phi(t_0) e^{1/(1 - e^{-m_\phi/T})}. \quad (11)$$

Now, since  $n_\phi(t_0)$  includes a statistical weight factor,  $e^{-m_\phi/T}$ , (we assume  $T \leq m_\phi$  so that it is approximately a Boltzmann weight), the growth in number density of inflatons begins to overwhelm the exponential suppression from the statistical weight when

$$T \approx m_\phi(1 - e^{-m_\phi/T}), \quad (12)$$

which is uniquely satisfied for  $T \approx 0.74m_\phi$ . We regard this as a maximum or limiting temperature, reminiscent of the Hagedorn temperature from hadronic physics. This is the main result of this work. Beyond this temperature, the radiation fields overlap strongly and populate the zero momentum mode of the inflaton at a rate that is sufficient to overcome the thermal suppression. The universe then has only an inflaton component to its energy density. We see that the analytical estimate of  $T_{\max}$  or  $T_H$  is close to the numerical value obtained from evolving the Boltzmann equations for inflaton decay backwards in time. This numerical estimate is robust and is not affected by inclusion of inverse reaction terms in the collision kernel of the Boltzmann equations, which only become significant at temperatures exceeding the limiting temperature. In [16], modulo extra assumptions on the gauge interactions of the fermions and their coupling to the gauge fields, it was argued that  $T_{\max}$  could exceed  $m_\phi$ . We find that in the Hagedorn picture, including thermal masses for the fermions, as in [16], does not affect the bound  $T_{\max} \approx 0.74m_\phi$ .

In summary, we have shown that it is possible to interpret the maximum temperature reached in the evolution of inflaton decay as a Hagedorn temperature. Viewed backwards in time, or with decreasing size of the universe, at this temperature, which is close to  $m_\phi$ , coherent population of the zero momentum state of the inflaton takes place. The rate of population exceeds the thermal suppression factor, and it is not appropriate to use the evolution equations any longer, since a clear distinction between radiation and inflaton fields cannot be made. This is similar to the purported hadron gas to quark-gluon plasma (QGP) transition at finite temperature  $T \sim m_\pi$ , where the Hagedorn temperature signifies when the quark fields start to percolate through the overlapping hadrons. At this point, hadrons are no longer genuine physical degrees of freedom, and the deconfinement phase transition is approached. We have applied the same idea to the early stage of inflaton decay, and supported the conjecture with a numerical analysis of the Boltzmann equations.

We find that the maximum temperature, or the corresponding Hagedorn temperature of the post-inflationary universe is very close to the inflaton mass,  $T_{\max} \approx 0.74m_\phi$ . We regard this as an actual reheat temperature of the universe because below the phase transition, the subsequent evolution of radiation density obeys an

adiabatic expansion law with  $T \propto a^{-1}$ . We consider this result as having an impact on particle-cosmology which we shall explore in a separate publication.

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